

Reply to ‘‘Comment on ‘Dynamics of wetting fronts in porous media’ ’’

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In our work [Phys. Rev. E **58**, R5245 (1998)] we introduced a dynamic phenomenological approach to model propagation of localized wetting fronts in porous media. Gray and Miller in their Comment [Phys. Rev. E **61**, 2150 (2000)] criticize our approach on several issues. The main criticism addresses the problem of mass conservation in our model. In this Reply we argue that their criticism is incorrect.

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In our paper [1] we applied a phase-field approach to model propagation of localized wetting fronts in porous media. Our work was motivated by numerous experimental pieces of evidence of a dynamic relation between capillary pressure and medium saturation. This dynamic relation is not reflected in the traditional models of flow in porous media (see references in [1]).

We are grateful to Gray and Miller (GM) [2] for their interest in our work. While acknowledging the importance of the subject, they criticize our approach on several accounts. In their Comment GM make two main points. The first regarding the particular form of the Richard’s equation (RE) is relatively minor, and does not relate to the model we have presented. The second is more serious. They claim that our model does not conserve mass. We address these issues below using the equation numbering of the Comment.

Before addressing GM’s criticism, we note that, contrary to the statement made by GM (see, e.g., abstract of the Comment), we do not present a ‘‘new method to model unsaturated flow.’’ Instead, we consider only a phenomenological description of localized wetting fronts. Such fronts separate homogeneously dry and wet regions of porous media, and are often modeled as *sharp interfaces* [3]. While obviously related, our problem differs from the general unsaturated flow study where such interfaces might not even exist.

As the first main point of the Comment, GM claim that our form of RE (5) ‘‘is not consistent’’ with their form of RE (4). However, our model is not in any way based on RE. The latter is mentioned only to demonstrate the shortcomings of the traditional approach and the consistency between the moving front solutions obtained by each of the approaches for incompressible media and fluids. Moreover, for that case, the difference between Eqs. (4) and (5) is simply a matter of rescaling time by factor of ω . This rescaling is allowed, since porosity ω is dimensionless and, in the incompressible case, constant.

We now proceed to the second main point of the Comment, i.e., mass conservation in our model. Our dynamic model consists of a system of two coupled equations for capillary pressure ψ and saturation θ , Eqs. (6) and (7), respectively. For incompressible fluids and media Eq. (6) reduces to Eq. (8) (see also [1]). GM question the validity of both Eqs. (8) and (7).

GM claim that, since (i) Eq. (8) ‘‘is not consistent with RE’’ [Eq. (4) or (5)], and (ii) RE conserves mass, then Eq. (8) does not conserve mass. This statement is somewhat surprising. Clearly, there is an abundance of equations which both conserve mass and are not consistent with RE. Note that, since RE ignores the dynamic relation between θ and ψ , we do not expect our model to be consistent with it. One can easily verify that Eq. (8) conserves mass, and the total mass is only changing due to an external mass flux. This can be done integrating the equation over the entire domain, and using constant flux boundary conditions for capillary pressure, and with saturation $\theta=1$ or 0, behind or ahead of the front respectively.

With regard to Eq. (7), we present the following arguments to demonstrate mass conservation. First, if the solution to the full system (8) and (7) exists, and since Eq. (8) conserves mass, then the solution conserves mass as well. Our one-dimensional analytic solution in [1] clearly confirms this point.

Second, beyond the localized interfacial region, $\theta=0$ or 1, so the nonlinear term in Eq. (7) vanishes. Thus no mass is produced or removed. Within the localized region, it is easy to see from (7) that the total mass is conserved, since the wetting front is propagating in a self-similar manner. Contrary to GM’s claim, mass in our model is neither generated, nor annihilated, even for nonzero last term in Eq. (7). Instead, it is redistributed within the localized interfacial zone due to nonlinear capillary effects. The particular form of this term has no rigorous physical motivation, since our description is phenomenological [1]. The obtained results justify our choice.

In addition, GM comment that in a steady state regime the equation for saturation (8) results in a θ - ψ relation which is independent of the medium and fluid characteristics. In fact, Eq. (8) by itself is insufficient to provide a solution to the problem and has to be closed by Eq. (7). Thus the solution to the steady state problem depends on such medium and fluid characteristics, as width of the moving front W , and the capillary pressure on the front, ψ_f (see the analytic solution in [1]).

GM also state that the existence of only two stable states of saturation, wet ($\theta=1$) and dry ($\theta=0$), is ‘‘overly restrictive.’’ In fact, the saturation in our model gradually varies between 1 and 0 within a localized interfacial region. It is

precisely such a situation, rather than a general unsaturated flow, that we are describing in our paper. The class of problems related to the propagation of localized interfaces is of general interest for readers of Phys. Rev. E.

We would like to conclude that our approach needs further work and modifications. This, however, does not diminish its main result, i.e., developing a dynamic model of wetting front propagation in porous media.

[1] I. Mitkov, D. M. Tartakovsky, and C. L. Winter, Phys. Rev. E **58**, R5245 (1998).

[2] W. G. Gray and C. T. Miller, preceding Comment, Phys. Rev.

E **61**, 2150 (2000).

[3] W. H. Green and G. A. Ampt, J. Agric. Sci. **4**, 1 (1911); G. L. Chen and S. P. Neuman, Phys. Fluids **8**, 353 (1996).